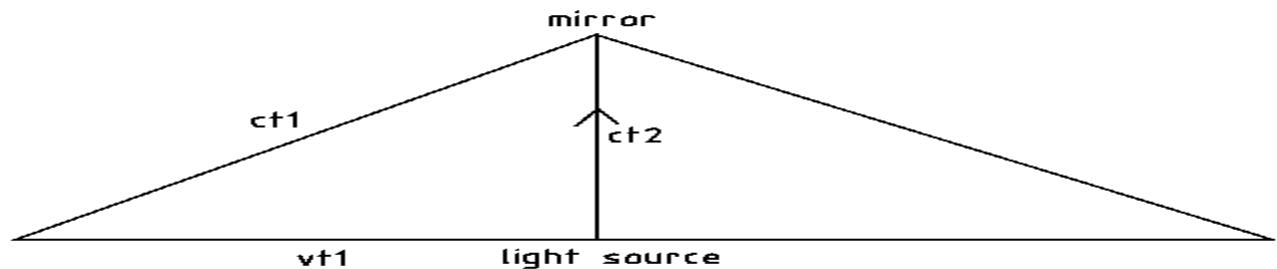


## Special Relativity

In 1905, Albert Einstein wrote a paper describing the effects of high speeds on time, distance, mass, momentum and energy.

He postulated that the only absolute in the universe is the speed of light in a vacuum,  $c = 3 \times 10^8$  m/s, regardless of the speed of the observer.



## Time Dilation

Imagine an astronaut on a spacecraft, shining a flashlight at a mirrored ceiling:

The distance travelled by the light for 1/2 the round trip as measured by the astronaut is  $d_a = ct_2$ . That measured by the observer on Earth is  $d_E = ct_1$ . The distance travelled by the ship during this time is  $d_s = vt_1$ .

“v” = the speed of the ship

Using pythagorean theorem,

$$(c \Delta t_1)^2 = (v \Delta t_1)^2 + (c \Delta t_2)^2$$

$$(c \Delta t_1)^2 - (v \Delta t_1)^2 = (c \Delta t_2)^2$$

$$c^2(\Delta t_1)^2 - v^2(\Delta t_1)^2 = c^2(\Delta t_2)^2$$

now divide by  $c^2$ :

$$(\Delta t_1)^2 - \frac{v^2}{c^2}(\Delta t_1)^2 = (\Delta t_2)^2$$

$$(\Delta t_1)^2 \left( 1 - \frac{v^2}{c^2} \right) = (\Delta t_2)^2$$

$$(\Delta t_1)^2 = \frac{(\Delta t_2)^2}{\left( 1 - \frac{v^2}{c^2} \right)}$$

$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where

$\Delta t_1$  = the time that passes for the stationary (Earth) observer

$\Delta t_2$  = the time that passes for the moving observer

Thus, time passes more slowly for faster moving objects!

### **Length (and Distance) Contraction**

We see a similar effect for distances:

$$L_1 = \frac{L_2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Where  $L_1$  = the rest length of the object (this is the same as the length of the object as measured by the observer moving with the object)

$L_2$  = the length of the object as measured by the stationary

observer

**\*\* NOTE** – This can also be used for distance travelled, where

$L_1$  = the distance travelled by the moving object as measured by the stationary observer

$L_2$  = the distance travelled by the moving object as measured by the moving object

### **Mass and Momentum Increase**

We also see a similar effect for mass and momentum:

$$m_1 = \frac{m_2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Where  $m_1$  = the “relativistic” mass of the object (the mass of the object while moving, as measured by the stationary observer)

$m_2$  = the “rest” mass of the object (the mass of the object while at rest)

so,

$$p = \frac{mv}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

Where “p” is the relativistic momentum of the object

Examples:

1. Jim and Tim are twins. Jim becomes a starship captain and goes on a five-year mission (as measured by him), travelling at 96% of the speed of light. At the end of the mission, what is the age difference between the twins and who is older?

$$\Delta t_1 = \frac{\Delta t_2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad \Delta t_1 = \text{time that passes for Tim}$$

$\Delta t_2$  = time that passes for Jim

$$\Delta t_1 = \frac{5}{\sqrt{\left(1 - \frac{(0.96c)^2}{c^2}\right)}}$$

$$\Delta t_1 = 17.8 \text{ years}$$

Therefore, Tim is  $(17.8 - 5 = 12.8)$  years older than Jim.

2. If the rest mass of Jim's ship is  $2.4 \times 10^5 \text{ kg}$ , what are its relativistic mass and momentum during the mission?

$$m_1 = \frac{2.4 \times 10^5}{\sqrt{(1 - (0.96)^2)}}$$

$$m_1 = 8.6 \times 10^5 \text{ kg}$$

$$p = \frac{mv}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$p = (8.6 \times 10^5)(0.96)(3 \times 10^8)$$

$$p = 2.47 \times 10^{14} \text{ kg} \cdot \text{m/s}$$